Columbia University
in the City of New York

DEPARTMENT OF CIVIL ENGINEERING
AND ENGINEERING MECHANICS

THE PHOTOELASTIC DETERMINATION OF STRESS ON TRANSVERSE PLANES OF SYMMETRY FOR THE GENERAL AXI-SYMMETRIC CASE

by

E. A. FOX

Office of Naval Research Project NR-064-388
Contract Nonr-266(09)
Technical Report No. 15
CU-16-54-ONR-266(09)-CE
June 1954

Columbia University
in the City of New York

DEPARTMENT OF CIVIL ENGINEERING
AND ENGINEERING MECHANICS

THE PHOTOELASTIC DETERMINATION OF STRESS ON TRANSVERSE PLANES OF SYMMETRY FOR THE GENERAL AXI-SYMMETRIC CASE

by

E. A. FOX

Office of Naval Research Project NR-064-388
Contract Nonr-266(09)
Technical Report No. 15
CU-16-54-ONR-266(09)-CE
June 1954

THE PHOTOELASTIC DETERMINATION OF STRESS ON TRANSVERSE PLANES OF SYMMETRY FOR THE GENERAL AXI-SYMMETRIC CASE

Abstract

The method of O'Rourke and Saenz of treating the gross retardation patterns of symmetrically strained cylinders and spheres as an Abel integral equation is combined with one scattered light measurement to provide a complete solution on transverse planes of symmetry for the general axi-symmetric problem. A simple expression is derived for three-dimensional "notch stresses."

I. Introduction

The standard three-dimensional photoelastic techniques: freezing-slicing, and scattered light probing, have intrinsic limitations. Slicing is destructive of the model, probing requires a multiplicity of measurements, and neither is easily adapted to dynamic loading.

The idea of determining the interior stresses from the integrated relative retardation pattern is tantalizing and has been pursued by several investigators. Poritsky[1] achieved a solution for cylindrical bars in a state of plane strain; Read,[2] a solution for cylindrical glass bulbs under restrictive conditions. Kammerer[3] established the integral equation for the relative retardation in the axi-symmetric case from Neumann's[4] equations.

---

[1] H. Poritsky, Physics 5, 406-411 (1934).

[2] W. T. Read, Jr., J. Appl. Phys. 21, 250-257 (1950).

[3] A. Kammerer, Recherches sur la photoelasticimetrie (Hermann et Cie., Paris, 1944) pp. 141, 157.

[4] F. E. Neumann, Abh. d. Kon. Akad. d. Wissenschaften zu Berlin (1841) Part II, p. 1.

O'Rourke and Saenz[5] observed that this equation is Abel's integral equation. They concerned themselves with axially symmetric plane strain in long cylinders and radially symmetric stress in spheres, but required a very restrictive sum rule, $\sigma_z = \sigma_r + \sigma_\theta$; in the case of cylinders to obtain a solution. This restriction was removed by Saenz[6] by interferometric data and by Drucker and Woodward[7] by the use of oblique incidence.

The following discussion will apply to the general axi-symmetric case on a plane of transverse symmetry of an elastic body.

II. Preliminary Equations

Let the normal to the wave front be parallel to the y -axis, then the Maxwell-Neumann stress optic law[8] relates the relative retardation δ , the principal stresses in the plane of the wave front p , q $(p>q)$, the orientation ϕ of q with respect to x , and γ , the arc tan of the amplitude ratio of the two transmitted waves, as follows:

$$\frac{\partial \delta(x,y,z)}{\partial y} + C(p-q) = 2\frac{\partial \phi}{\partial y} \cot 2\gamma \sin \delta$$

$$\frac{\partial \gamma}{\partial y} = -\frac{\partial \phi}{\partial y} \cos \delta \qquad (1)$$

where C is the stress-optic coefficient.

---

[5] R. C. O'Rourke and A. W. Saenz, Quart. Appl. Math. 8, 303-311 (1950). R. C. O'Rourke, J. Appl. Phys. 22, 872-878 (1951).
[6] A. W. Saenz, J. Appl. Phys. 21, 962-965 (1950).
[7] D. C. Drucker and W. B. Woodward, J. Appl. Phys. 25, 510-512 (1954).
[8] E. G. Coker and L. N. G. Filon, A Treatise on Photo-Elasticity (Cambridge University Press, Cambridge, 1931) p. 256.

Let the axis of symmetry be z. Then, in a usual notation, the following relations[9] hold:

$$\frac{\partial \sigma_r}{\partial r} + \frac{\partial \tau_{rz}}{\partial z} + \frac{\sigma_r - \sigma_\theta}{r} = 0 \qquad (2)$$

$$\frac{\partial \sigma_z}{\partial z} + \frac{\partial \tau_{rz}}{\partial r} + \frac{1}{r}\tau_{rz} = 0 \qquad (3)$$

$$\frac{\partial u}{\partial r} = \epsilon_r = \frac{1}{E}\left[\sigma_r - \nu(\sigma_\theta + \sigma_z)\right] \qquad (4)$$

$$\frac{u}{r} = \epsilon_\theta = \frac{1}{E}\left[\sigma_\theta - \nu(\sigma_r + \sigma_z)\right] \qquad (5)$$

where u is the displacement in the r direction. Eliminating u between (4) and (5) we obtain:

$$\frac{\sigma_r - \sigma_\theta}{r} = \frac{1}{1+\nu}\frac{\partial}{\partial r}\left[\sigma_\theta - \nu(\sigma_r + \sigma_z)\right] = \frac{E}{1+\nu}\frac{\partial}{\partial r}\left(\frac{u}{r}\right) \qquad (6)$$

In the plane of the wave front

$$p - q = \left[\left(\sigma_r \frac{x^2}{r^2} + \sigma_\theta \frac{y^2}{r^2} - \sigma_z\right)^2 + 4\tau_{rz}^2 \frac{x^2}{r^2}\right]^{\frac{1}{2}} \qquad (7)$$

$$\sin 2\phi = \frac{2\tau_{rz}}{p-q}\frac{x}{r} \qquad (8)$$

III. Transverse Plane of Symmetry

Let $z = 0$ be a transverse plane of symmetry. Let a, b be the inner and outer radii, respectively, of the section of the body cut by $z = 0$. On $z = 0$, $\tau_{rz} = 0$. Hence, from (8), $\phi = 0$: therefore (1) and (7) become

---

[9] A. E. H. Love, Mathematical Theory of Elasticity, 4th Ed., (Dover Publications, New York, 1944) p. 274.

$$\frac{\partial \delta(x,y,0)}{\partial y} = -C(p-q)_{z=0} = C\left[\sigma_z - \sigma_r \frac{x^2}{r^2} - \sigma_\theta \frac{y^2}{r^2}\right]_{z=0} \tag{9}$$

Consider a pencil of circularly polarized light along the path $x = 0$. Then (9) becomes

$$C(\sigma_z - \sigma_\theta)_{z=0} = \frac{\partial \delta(0,y,0)}{\partial y} = \frac{\partial \delta(r,0)}{\partial r} = S(r) \tag{10}$$

where $S(r)$ is Weller's[10] scattered light function which is inversely proportional to the spacing of the interference fringes viewed normally to the light path.

Let $R(x,z)$ be the two-dimensional map of the integrated relative retardation. Put (6) and (10) in (9) and integrate across the chord with respect to y. Let $t = [b^2 - x^2]^{\frac{1}{2}}$ be the half chord length. Then since all functions are even in y

$$R(x,0) = 2\int_0^t S(r)\,dy - 2C\frac{E}{1+\nu}x^2\int_0^t \frac{1}{r}\frac{d}{dr}\left(\frac{u(r,0)}{r}\right)dy$$

Changing the variable of integration to r and transposing, there results

$$\frac{1+\nu}{2CE}\frac{1}{x^2}\left\{2\int_x^b \frac{r\,S(r)}{\sqrt{r^2-x^2}}\,dr - R(x,0)\right\} = \int_x^b \frac{\frac{d}{dr}\left(\frac{u(r,0)}{r}\right)}{\sqrt{r^2-x^2}}\,dr \tag{11}$$

This is Abel's integral equation,[11] which, since the left hand side of (11) vanishes at $x = b$, has the unique continuous inverse

$$\frac{d}{dr}\left(\frac{u(r,0)}{r}\right) = \frac{1+\nu}{\pi EC}\frac{d}{dr}\int_r^b \frac{R(x,0) - 2\int_x^b \frac{r\,S(r)}{\sqrt{r^2-x^2}}\,dr}{x\sqrt{x^2-r^2}}\,dx = \frac{1+\nu}{\pi EC}\frac{d}{dr}M(r) \tag{12}$$

---

[10] R. Weller, Nat. Adv. Comm. Aero. Tech. Note 737, 1939.
[11] E. T. Whittaker and G. N. Watson, Modern Analysis (Cambridge University Press, New York, 1945) p. 229.

where $M(r)$ is an experimentally determined function. Integrate (12) with respect to r. Then, since $M(b) = 0$,

$$u(r,0) = r\left[\frac{1+\nu}{\pi E C} M(r) + \frac{u(b,0)}{b}\right] \tag{13}$$

where $u(b,0)$ is determined by measurement or is computed. (See Section IV.)

Sum (4) and (5), yielding

$$\sigma_r + \sigma_\theta = \frac{1}{1+\nu}\left[E\frac{1}{r}\frac{\partial}{\partial r}(ru) + 2\nu\sigma_z\right] \tag{14}$$

Finally, solving (6), (10), and (14)

$$\sigma_z(r,0) = \frac{E}{(1+\nu)(1-2\nu)}\left[\frac{u(r,0)}{r} + \nu\frac{du(r,0)}{dr}\right] + \frac{1-\nu}{C(1-2\nu)}S(r) \tag{15}$$

$$\sigma_\theta(r,0) = \frac{E}{(1+\nu)(1-2\nu)}\left[\frac{u(r,0)}{r} + \nu\frac{du(r,0)}{dr}\right] + \frac{\nu}{C(1-2\nu)}S(r) \tag{16}$$

$$\sigma_r(r,0) = \frac{E}{(1+\nu)(1-2\nu)}\left[2\nu\frac{u(r,0)}{r} + (1-\nu)\frac{du(r,0)}{dr}\right] + \frac{\nu}{C(1-2\nu)}S(r) \tag{17}$$

where $u(r)$ is given by (13), and $S(r)$ by (10).

IV. Determination of $u(b,0)$

$u(b,0)$ may either be measured or computed as follows: Put (2) in (9) and integrate with respect to y.

$$\frac{1}{2C}R(x,0) = \int_0^t \left[\sigma_z - \sigma_r - \frac{\partial\sigma_r}{\partial r}\frac{y^2}{r} - \frac{\partial\tau_{rz}}{\partial z}\frac{y^2}{r}\right]_{z=0} dy$$

Integrate the second term by parts, then after some manipulation (See Appendix.)

$$\frac{1}{2C}R(x,0) = \int_x^b \frac{\sigma_z(r,0) - \sigma_r(b,0) - \int_r^b \frac{\partial\tau_{rz}(r,0)}{\partial z}dr}{\sqrt{r^2-x^2}}\, r\, dr \tag{18}$$

This is again Abel's integral equation with the unique inverse

$$\sigma_z(r,o) = \sigma_r(b,o) - \frac{1}{\pi C}\frac{1}{r}\frac{d}{dr}\int_r^b \frac{R(x,o)}{\sqrt{x^2-r^2}}\, x\,dx + \int_r^b \frac{\partial \tau_{rz}(r,o)}{\partial z}\,dr \tag{19}$$

Put (2) in (6), then

$$\frac{\partial}{\partial r}(\sigma_r + \sigma_\theta) = \nu\frac{\partial \sigma_z}{\partial r} - (1+\nu)\frac{\partial \tau_{rz}}{\partial z} \tag{20}$$

Integrate (20) with respect to r. Then this with (5), (14), and (19) yields

$$E\frac{1}{r}\frac{d}{dr}\left[r\,u(r,o)\right] = E(1-\nu)\frac{u(b,o)}{b} + (1-\nu^2)\frac{1}{\pi C}\frac{1}{r}\frac{d}{dr}\int_r^b \frac{R(x,o)}{\sqrt{x^2-r^2}}\, x\,dx$$

$$+ (1+\nu)(1-2\nu)\,\sigma_z(r,o)$$

or, integrating,

$$b\,u(b,o) - a\,u(a,o) = \frac{1-\nu}{2}\frac{u(b,o)}{b}(b^2-a^2) - \frac{1-\nu^2}{\pi E C}\int_a^b \frac{R(x,o)}{\sqrt{x^2-a^2}}\, x\,dx$$

$$+ \frac{(1+\nu)(1-2\nu)}{E}\frac{P}{2\pi} \tag{21}$$

where $P = 2\pi\int_a^b \sigma_z(r,o)\,r\,dr$ = total axial force across $z = o$.

Put (13) in (21), obtaining

$$u(b,o) = \frac{2b}{(1+\nu)(b^2-a^2)}\left[a^2\frac{1+\nu}{\pi E C}M(a) - \frac{1-\nu^2}{\pi E C}\int_a^b \frac{R(x,o)}{\sqrt{x^2-a^2}}\,x\,dx + \frac{(1+\nu)(1-2\nu)}{E}\frac{P}{2\pi}\right] \tag{22}$$

If in particular $a = o$, then

$$u(b,o) = \frac{1}{b\pi E}\left[(1-2\nu)P - \frac{2(1-\nu)}{C}\int_0^b R(x,o)\,dx\right] \tag{22'}$$

V. Alternate Expression for $\sigma_z(b,0)$

If it is desired to obtain only the axial stress on the outer boundary of the section $z=0$, then a simple expression may be derived directly from the Maxwell-Neumann law. Consider (9). Let $x \to b$. Then $r \to b$, $t \to y \to 0$, $\delta(x,y,0) \to \delta(b,0,0) = \frac{1}{2} R(b,0)$. Replot $R(x,0)$ as $R(t,0)$ where $t = [b^2 - x^2]^{\frac{1}{2}}$. Then in the limit

$$\sigma_z(b,0) = \frac{1}{2C}\left[\frac{dR(t,0)}{dt}\right]_{t=0} + \sigma_r(b,0) \tag{23}$$

where $\sigma_r(b,0)$ = the normal surface traction at $(b,0)$. Thus $\sigma_z(b,0)$ is proportional to the gradient at the boundary of the integrated relative retardation reckoned as a function of the half light path. We observe that (23) is consistent with (19) if R is made a function of t. Further, if $u(b,0)$ is known, then (5) and (23) yield

$$\sigma_\theta(b,0) = E\frac{u(b,0)}{b} + \nu\left\{2\sigma_r(b,0) + \frac{1}{2C}\left[\frac{dR}{dt}(t,0)\right]_{t=0}\right\} \tag{24}$$

Acknowledgment

The author wishes to thank Professor R. D. Mindlin for suggesting this investigation and for his advice during its course.

Appendix

Derivation of Equation (18)

Integrate (9) with respect to y.

$$\frac{1}{2c} R(x,0) = \int_0^t \left[\sigma_z - \sigma_r \frac{x^2}{r^2} - \sigma_\theta \frac{y^2}{r^2}\right]_{z=0} dy = \int_0^t \left[(\sigma_z - \sigma_r) + (\sigma_r - \sigma_\theta)\frac{y^2}{r^2}\right]_{z=0} dy$$

Then, with (2),

$$\frac{1}{2c} R(x,0) = \int_0^t \left[\sigma_z - \sigma_r - \left(\frac{\partial \sigma_r}{\partial r} + \frac{\partial \tau_{rz}}{\partial z}\right)\frac{y^2}{r}\right]_{z=0} dy$$

Integrate the second term by parts, yielding

$$\frac{1}{2c} R(x,0) = \int_0^t \left[\sigma_z - \frac{\partial \tau_{rz}}{\partial z}\frac{y^2}{r}\right]_{z=0} dy - \Big[y\,\sigma_r(r,0)\Big]_0^t + \int_0^t \left[y \frac{\partial \sigma_r}{\partial r}\frac{dr}{dy}\right]_{z=0} dy$$

$$- \int_0^t \left[\frac{\partial \sigma_r}{\partial r}\frac{y^2}{r}\right]_{z=0} dy$$

but $\frac{dr}{dy} = \frac{y}{r}$ and $\Big[y\,\sigma_r(r,0)\Big]_0^t = \int_0^t \sigma_r(b,0)\,dy$, hence

$$\frac{1}{2c} R(x,0) = \int_0^t \left[\sigma_z - \sigma_r(b,0) - \frac{\partial \tau_{rz}}{\partial z}\frac{y^2}{r}\right]_{z=0} dy$$

Consider

$$\int_0^t \frac{\partial \tau_{rz}}{\partial z}\frac{y^2}{r}\,dy = \int_0^t \left\{\frac{d}{dr}\int_b^r \frac{\partial \tau_{rz}}{\partial z}dr\right\}\frac{y^2}{r}\,dy = \int_0^t \frac{r}{y}\left\{\frac{d}{dy}\int_b^r \frac{\partial \tau_{rz}}{\partial z}dr\right\}\frac{y^2}{r}\,dy$$

$$= \left[y\int_b^r \frac{\partial \tau_{rz}}{\partial z}dr\right]_0^t - \int_0^t \left\{\int_b^r \frac{\partial \tau_{rz}}{\partial z}dr\right\} dy$$

Thus

$$\frac{1}{2c} R(x,0) = \int_0^t \left[\sigma_z - \sigma_r(b,0) - \int_r^b \frac{\partial \tau_{rz}}{\partial z}dr\right]_{z=0} dy$$

Then, changing the variable of integration to r, we obtain (18).

DISTRIBUTION LIST

for

Technical and Final Reports Issued Under
Office of Naval Research Project NR-064-388. Contract Nonr-266(09)

Administrative, Reference and Liaison Activities of ONR

Chief of Naval Research
Department of the Navy
Washington 25, D.C.
Attn: Code 438 (2)
Code 416 (1)
Code 421 (1)

Director, Naval Research Lab.
Washington 25, D.C.
Attn: Tech. Info. Officer (9)
Technical Library (1)
Mechanics Division (2)
Code 3834 (J. P. Walsh) (1)

Commanding Officer
Office of Naval Research
Branch Office
150 Causeway Street
Boston 10, Massachusetts (1)

Commanding Officer
Office of Naval Research
Branch Office
346 Broadway
New York 13, New York (1)

Commanding Officer
Office of Naval Research
Branch Office
844 N. Rush Street
Chicago 11, Illinois (1)

Commanding Officer
Office of Naval Research
Branch Office
1000 Geary Street
San Francisco 24, California (1)

Commanding Officer
Office of Naval Research
Branch Office
1030 Green Street
Pasadena, California (1)

Contract Administrator, SE Area
Office of Naval Research
c/o George Washington University
707 22nd Street, N.W.
Washington 6, D.C. (1)

Officer in Charge
Office of Naval Research
Branch Office, London
Navy No. 100
FPO, New York, N.Y. (5)

Library of Congress
Washington 25, D.C.
Attn: Navy Research Section (2)

Department of Defense
Other Interested Government Activities

GENERAL

Research and Development Board
Department of Defense
Pentagon Building
Washington 25, D.C.
Attn: Library (Code 3D-1075) (1)

Armed Forces Special Weapons Project
P.O. Box 2610
Washington, D.C.
Attn: Col. G. F. Blunda (1)

ARMY

Chief of Staff
Department of the Army
Research and Development Division
Washington 25, D.C.
Attn: Chief of Res. and Dev. (1)

Office of the Chief of Engineers
Assistant Chief for Works
Department of the Army
Bldg. T-7, Gravelly Point
Washington 25, D.C.
Attn: Structural Branch
(R. L. Bloor) (1)

Office of the Chief of Engineers
Asst. Chief for Military Construction
Department of the Army
Bldg. T-7, Gravelly Point
Washington 25, D.C.
Attn: Structures Branch
(H. F. Carey) (1)

Engineering Research & Development Lab.
Fort Belvoir, Virginia
Attn: Structures Branch (1)

The Commanding General
Sandia Base, P.O. Box 5100
Albuquerque, New Mexico
Attn: Col. Canterbury (1)

Operations Research Officer
Department of the Army
Ft. Lesley J. McNair
Washington 25, D.C.
Attn: Howard Brackney (1)

Office of Chief of Ordnance
Research & Development Service
Department of the Army
The Pentagon
Washington 25, D.C.
Attn: ORDTB (2)

Commanding Officer
Ballistic Research Laboratory
Aberdeen Proving Ground
Aberdeen, Maryland
Attn: Dr. C. W. Lampson (1)

ARMY (cont.)

Commanding Officer
Watertown Arsenal
Watertown, Massachusetts
Attn: Laboratory Division (1)

Commanding Officer
Frankford Arsenal
Philadelphia, Pennsylvania
Attn: Laboratory Division (1)

Commanding Officer
Squier Signal Laboratory
Fort Monmouth, New Jersey
Attn: Components and Materials
Branch (1)

NAVY

Chief of Bureau of Ships
Navy Department
Washington 25, D.C.
Attn: Director of Research (2)

Director
David Taylor Model Basin
Washington 7, D.C.
Attn: Structural Mechanics Div. (2)

Director
Naval Engr. Experiment Station
Annapolis, Maryland (1)

Director
Materials Laboratory
New York Naval Shipyard
Brooklyn 1, New York (1)

Chief of Bureau of Ordnance
Navy Department
Washington 25, D.C.
Attn: Ad-3, Technical Library (1)

Superintendent
Naval Gun Factory
Washington 25, D.C. (1)

Naval Ordnance Laboratory
White Oak, Maryland
RFD 1, Silver Spring, Maryland
Attn: Mechanics Division (2)

Naval Ordnance Test Station
Inyokern, California
Attn: Scientific Officer (1)

NAVY (cont.)

Commander, U.S. N.O.T.S.
Pasadena Annex
3202 E. Foothill Blvd.
Pasadena 8, California
Attn: Code P5507 (1)

Commander, U.S. N.O.T.S.
China Lake, California
Attn: Code 501 (1)

Chief of Bureau of Aeronautics
Navy Department
Washington 25, D.C.
Attn: TD-41, Technical Library (1)

Naval Air Experimental Station
Naval Air Materiel Center
Naval Base
Philadelphia 12, Pennsylvania
Attn: Head, Aeronautical Materials Laboratory (1)

Chief of Bureau of Yards & Docks
Navy Department
Washington 25, D.C.
Attn: Code P-314 (1)

Officer in Charge
Naval Civil Engr. Research and Eval. Laboratory
Naval Station
Port Hueneme, California (1)

Commander
U.S. Naval Proving Grounds
Dahlgren, Virginia (1)

AIR FORCES

Commanding General
U.S. Air Forces
The Pentagon
Washington 25, D.C.
Attn: Research & Development Division (1)

Commanding General
Air Materiel Command
Wright-Patterson Air Force Base
Dayton, Ohio
Attn: MCREX-B (E. H. Schwartz) (1)

AIR FORCES (cont.)

Office of Air Research
Wright-Patterson Air Force Base
Dayton, Ohio
Attn: Chief, Applied Mechanics Group (1)

OTHER GOVERNMENT AGENCIES

U.S. Atomic Energy Commission
Division of Research
Washington, D.C. (1)

Argonne National Laboratory
P.O. Box 5207
Chicago 80, Illinois (1)

Director
National Bureau of Standards
Washington, D.C.
Attn: Dr. W. H. Ramberg (1)

U.S. Coast Guard
1300 E Street, N.W.
Washington, D.C.
Attn: Chief, Testing & Developing Division (1)

Forest Products Laboratory
Madison, Wisconsin
Attn: L. J. Markwardt (1)

National Advisory Committee for Aeronautics
1724 F Street, N.W.
Washington, D.C. (1)

National Advisory Committee for Aeronautics
Langley Field, Virginia
Attn: Dr. E. Lundquist (1)

National Advisory Committee for Aeronautics
Cleveland Municipal Airport
Cleveland, Ohio
Attn: J. H. Collins, Jr. (1)

U.S. Maritime Commission
Technical Bureau
Washington, D.C.
Attn: V. Russo (1)

Contractors and Other Investigators
Actively Engaged in Related Research

Professor J. R. Andersen
Towne School of Engineering
University of Pennsylvania
Philadelphia, Pennsylvania (1)

Professor Melvin Baron
Dept. of Civil Engineering
Columbia University
New York 27, New York (1)

Professor Lynn Beedle
Fritz Engineering Laboratory
Lehigh University
Bethlehem, Pennsylvania (1)

Professor C. B. Biezeno
Technische Hoogeschool
Nieuwe Laan 76
Delft, Holland (1)

Dr. M. A. Biot
1819 Broadway
New York, New York (1)

Professor R. L. Bisplinghoff
Dept. of Aeronautical Engineering
Massachusetts Institute of Technology
Cambridge 39, Massachusetts (1)

Professor Hans H. Bleich
Dept. of Civil Engineering
Columbia University
New York 27, New York (1)

Professor J. A. Bogdanoff
Purdue University
Lafayette, Indiana (1)

Professor B. A. Boley
Dept. of Civil Engineering
Columbia University
New York 27, New York (1)

Professor P. W. Bridgeman
Dept. of Physics
Harvard University
Cambridge 38, Massachusetts (1)

Professor D. M. Burmister
Dept. of Civil Engineering
Columbia University
New York 27, New York (1)

Dr. V. Cadambe
Assistant Director of the National
Physical Laboratory of India
Hillside Road
New Delhi 12, India (1)

Professor George F. Carrier
Division of Applied Science
Pierce Hall
Harvard University
Cambridge 38, Massachusetts (1)

Dr. David Cheng
M. W. Kellogg Company
225 Broadway
New York, New York (1)

Committee on Government Aided Research
Columbia University
313 Low Memorial Library
New York 27, New York (2)

Mrs. Hilda Cooper
The Dell
Searingtown
Albertson, Long Island, New York (1)

Dr. Antoine E. I. Craya
Neyrpic
Boite Postale 52
Grenoble, France (1)

Professor J. P. Den Hartog
Massachusetts Institute of Technology
Cambridge 39, Massachusetts (1)

Professor Herbert Deresiewicz
Dept. of Civil Engineering
Columbia University
632 West 125th Street
New York 27, New York (1)

Dr. C. C. Dohrenwend
Rensselaer Polytechnic Institute
Troy, New York (1)

Professor T. J. Dolan
Dept. of Theoretical and Applied
Mechanics
University of Illinois
Urbana, Illinois (1)

Contractors and Other Investigators Actively Engaged in Related Research (cont.)

Professor Lloyd Donnell
Dept. of Mechanics
Illinois Institute of Technology
Chicago 16, Illinois (1)

Professor D. C. Drucker
Division of Engineering
Brown University
Providence 12, Rhode Island (1)

Dr. W. Eckert
Watson Scientific Computing Laboratory
612 West 116th Street
New York 27, New York (1)

Dr. H. Ekstein
Armour Research Foundation
Illinois Institute of Technology
Chicago 16, Illinois (1)

Engineering Library
Columbia University
New York 27, New York (1)

Professor E. L. Eriksen
University of Michigan
Ann Arbor, Michigan (1)

Professor A. C. Eringen
Illinois Institute of Technology
Chicago 16, Illinois (1)

Dr. W. L. Esmeijer
Voorduinstraat 24
Haarlem, Holland (1)

Mr. Marvin Forray
1396 East 16th Street
Brooklyn 30, New York (1)

Dr. F. Forscher
Westinghouse Atomic Power Division
P.O. Box 1468
Pittsburgh 30, Pennsylvania (1)

Professor A. M. Freudenthal
Dept. of Civil Engineering
Columbia University
New York 27, New York (1)

Professor B. Fried
Washington State College
Pullman, Washington (1)

Professor K. O. Friedrichs
New York University
Washington Square
New York, New York (1)

Professor M. M. Frocht
Illinois Institute of Technology
Chicago 16, Illinois (1)

Professor J. M. Garrelts
Dept. of Civil Engineering
Columbia University
New York 27, New York (1)

Professor J. A. Goff
University of Pennsylvania
Philadelphia, Pennsylvania (1)

Mr. E. A. Gerber
Signal Corps Engineering Labs.
Fort Monmouth, New Jersey
Watson Area (1)

Mr. Martin Goland
Midwest Research Institute
4049 Pennsylvania
Kansas City 2, Missouri (1)

Dr. J. N. Goodier
Dept. of Engineering Mechanics
Stanford University
Stanford, California (1)

Professor L. E. Goodman
Dept. of Mechanical Engineering
University of Minnesota
Minneapolis 14, Minnesota (1)

Professor R. J. Hansen
Massachusetts Institute of Technology
Cambridge 39, Massachusetts (1)

Professor R. M. Hermes
University of Santa Clara
Santa Clara, California (1)

Professor G. Herrmann
Dept. of Civil Engineering
Columbia University
New York 27, New York (1)

Contractors and Other Investigators Actively Engaged in Related Research (cont.)

Professor M. Hetényi
Northwestern University
Evanston, Illinois (1)

Professor T. J. Higgins
Dept. of Electrical Engineering
University of Wisconsin
Madison 6, Wisconsin (1)

Professor N. J. Hoff
Dept. of Aeronautical Engineering
Polytechnic Institute of Brooklyn
99 Livingston Street
Brooklyn 2, New York (1)

Professor M. B. Hogan
University of Utah
Salt Lake City, Utah (1)

Professor D. L. Holl
Iowa State College
Ames, Iowa (1)

Dr. J. H. Holiomon
General Electric Research Labs.
1 River Road
Schenectady, New York (1)

Professor W. H. Hoppmann
Dept. of Applied Mechanics
The Johns Hopkins University
Baltimore, Maryland (1)

Dr. Gabriel Horvay
Knolls Atomic Power Laboratory
General Electric Company
Schenectady, New York (1)

Institut de Mathématiques
Université
post. fah 55
Skoplje, Yugoslavia (1)

Professor L. S. Jacobsen
Dept. of Mechanical Engineering
Stanford University
Stanford, California (1)

Professor Bruce G. Johnston
University of Michigan
Ann Arbor, Michigan (1)

Professor Thomas R. Kane
25-2 Valley Road
Drexel Hill, Pennsylvania (1)

Professor K. Klotter
Stanford University
Stanford, California (1)

Professor W. J. Krefeld
Dept. of Civil Engineering
Columbia University
New York 27, New York (1)

Professor B. J. Lazan
Dept. of Materials Engineering
University of Minnesota
Minneapolis 14, Minnesota (1)

Professor E. H. Lee
Division of Applied Mathematics
Brown University
Providence 12, Rhode Island (1)

Professor George Lee
Rensselaer Polytechnic Institute
Troy, New York (1)

Professor J. M. Lessells
Dept. of Mechanical Engineering
Massachusetts Institute of Technology
Cambridge 39, Massachusetts (1)

Library, Engineering Foundation
29 West 39th Street
New York, New York (1)

Professor Paul Lieber
Dept. of Engineering
Rensselaer Polytechnic Institute
Troy, New York (1)

Dr. Hsu Lo
Purdue University
Lafayette, Indiana (1)

Professor C. T. G. Looney
Dept. of Civil Engineering
Yale University
New Haven, Connecticut (1)

Contractors and Other Investigators Actively Engaged in Related Research (cont.)

Dr. J. L. Lubkin
Midwest Research Institute
4049 Pennsylvania
Kansas City 2, Missouri (1)

Professor J. F. Ludloff
School of Aeronautics
New York University
New York 53, New York (1)

Professor J. N. Macduff
Rensselaer Polytechnic Institute
Troy, New York (1)

Professor C. W. MacGregor
University of Pennsylvania
Philadelphia, Pennsylvania (1)

Professor Lawrence E. Malvern
Dept. of Mathematics
Carnegie Institute of Technology
Pittsburgh 13, Pennsylvania (1)

Professor J. H. Marchant
Brown University
Providence 12, Rhode Island (1)

Professor J. Marin
Pennsylvania State College
State College, Pennsylvania (1)

Dr. W. P. Mason
Bell Telephone Laboratories
Murray Hill, New Jersey (1)

Professor R. D. Mindlin
Dept. of Civil Engineering
Columbia University
632 West 125th Street
New York 27, New York (15)

Dr. A. Nadai
136 Cherry Valley Road
Pittsburgh 21, Pennsylvania (1)

Professor Paul M. Naghdi
Dept. of Engineering Mechanics
University of Michigan
Ann Arbor, Michigan (1)

Professor N. M. Newmark
207 Talbot Laboratory
University of Illinois
Urbana, Illinois (1)

Professor Jesse Ormondroyd
University of Michigan
Ann Arbor, Michigan (1)

Professor W. Osgood
Illinois Institute of Technology
Chicago 16, Illinois (1)

Dr. George B. Pegram
313 Low Memorial Library
Columbia University
New York 27, New York (1)

Dr. R. P. Petersen
Director, Applied Physics Division
Sandia Laboratory
Albuquerque, New Mexico (1)

Mr. R. E. Peterson
Westinghouse Research Laboratories
East Pittsburgh, Pennsylvania (1)

Professor A. Phillips
School of Engineering
Stanford University
Stanford, California (1)

Professor Gerald Pickett
Dept. of Mechanics
University of Wisconsin
Madison 6, Wisconsin (1)

Dr. H. Poritsky
General Engineering Laboratory
General Electric Company
Schenectady, New York (1)

Professor W. Prager
Graduate Division of Applied Mathematics
Brown University
Providence 12, Rhode Island (1)

Dr. Frank Press
Lamont Geological Observatory
Palisades, New York (1)

Contractors and Other Investigators Actively Engaged in Related Research (cont.)

RAND Corporation
1500 4th Street
Santa Monica, California
Attn: Dr. D. L. Judd (1)

Dr. S. Raynor
Armour Research Foundation
Illinois Institute of Technology
Chicago 16, Illinois (1)

Professor E. Reissner
Dept. of Mathematics
Massachusetts Institute of Technology
Cambridge 39, Massachusetts (1)

Professor H. Reissner
Polytechnic Institute of Brooklyn
99 Livingston Street
Brooklyn 2, New York (1)

Dr. Kenneth Robinson
Combustion Engineering, Inc.
200 Madison Avenue
New York 16, New York (1)

Professor Leif Rongved
Dept. of Engineering Mechanics
Pennsylvania State College
State College, Pennsylvania (1)

Professor M. A. Sadowsky
Dept. of Mechanics
North Hall
Rensselaer Polytechnic Institute
Troy, New York (1)

Professor M. G. Salvadori
Dept. of Civil Engineering
Columbia University
New York 27, New York (1)

Mr. Arnold Schacknow
20-35 Seagirt Boulevard
Far Rockaway, New York (1)

Dr. F. S. Shaw
Superintendent
Structures & Materials Division
Aeronautical Research Laboratories
Box 4331, G.P.O. Melbourne
Victoria, Australia (1)

Dr. Daniel T. Sigley
American Machine and Foundry Company
511 Fifth Avenue
New York, New York (1)

Professor C. B. Smith
Department of Mathematics
Walker Hall
University of Florida
Gainesville, Florida (1)

Professor C. R. Soderberg
Dept. of Mechanical Engineering
Massachusetts Institute of Technology
Cambridge 39, Massachusetts (1)

Professor R. V. Southwell
Imperial College of Science and Technology
South Kensington
London S.W. 7, England (1)

Professor E. Sternberg
Illinois Institute of Technology
Chicago 16, Illinois (1)

Professor J. J. Stoker
New York University
Washington Square
New York, New York (1)

Mr. R. A. Sykes
Bell Telephone Laboratories
Murray Hill, New Jersey (1)

Professor P. S. Symonds
Brown University
Providence 12, Rhode Island (1)

Professor J. L. Synge
Dublin Institute for Advanced Studies
School of Theoretical Physics
64-65 Merrion Square
Dublin, Ireland (1)

Professor F. K. Teichmann
Dept. of Aeronautical Engineering
New York University
University Heights, Bronx
New York, New York (1)

Contractors and Other Investigators Actively Engaged in Related Research (cont.)

Professor S. P. Timoshenko
School of Engineering
Stanford University
Stanford, California (1)

Professor C. A. Truesdell
Graduate Institute for Applied Mathematics
Indiana University
Bloomington, Indiana (1)

Professor Karl S. Van Dyke
Department of Physics
Scott Laboratory
Wesleyan University
Middletown, Connecticut (1)

Dr. I. Vigness
Naval Research Laboratory
Anacostia Station
Washington, D.C. (1)

Dr. Leonardo Villena
Av. de La Habana. 147
Madrid, Spain (1)

Professor E. Volterra
Rensselaer Polytechnic Institute
Troy, New York (1)

Mr. A. M. Wahl
Westinghouse Research Laboratories
East Pittsburgh, Pennsylvania (1)

Professor C. T. Wang
Dert. of Aeronautical Engineering
New York University
University Heights, Bronx
New York, New York (1)

Dr. R. L. Wegel
RFD 2
Peekskill, New York (1)

Professor E. E. Weibel
University of Colorado
Boulder, Colorado (1)

Professor Jerome Weiner
Dept. of Civil Engineering
Columbia University
New York 27, New York (1)

Professor Alexander Weinstein
Institute of Applied Mathematics
University of Maryland
College Park, Maryland (1)

Professor Dana Young
Yale University
Winchester Hall
15 Prospect Street
New Haven, Connecticut (1)